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Comments on "Interference-based Capacity Analysis of Vehicular Ad Hoc Networks"

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***Abstract*—A new effective interference-based capacity model was proposed for VANET safety message broadcast scenario in letter [1]. This letter is a reconsideration and extension of the model in [1]. First, we point out that the analysis in [1] is incomplete and show a new derivation of the node transmission probability and the SIR distribution accounting for the impact of asynchronous timing of hidden terminals and all possible interference occurrence cases. Second, the analysis is extended to the derivation of other important quality of service (QoS) metrics such as packet reception probability, packet reception ratio, and transmission capacity, etc. The proposed analysis is cross validated by MATLAB, PYTHON, and NS2 simulations.**

***Index Terms*—Broadcast, Vehicular ad hoc networks, Interference, Capacity, Quality of Service.**

I. Introduction

Vehicular ad hoc networks (VANETs) have been proposed and studied for safety applications. Many investigations are on capacity and quality of service (QoS) of current vehicular communication systems such as Dedicated Short Range Communications (DSRC) regarding whether or not the systems are able to meet the stringent QoS requirements for the safety applications. Most analyses of VANET QoS and capacity assumed fixed communication range [2]–[4], which is not practical. Several studies for VANET capacity using scaling-law based method can only give per-node capacity scales in an asymptotically large and uniformly Poisson wireless networks under Rayleigh fading channel [5]–[9], which cannot be easily applied to actual network capacity estimation. Recently, a new interference-based capacity model was proposed for VANET safety message broadcast scenario [1]. The model approached the capacity analysis of one-dimensional (1-D) VANET safety message broadcast under *Nakagami* fading channel through derivation of signal-to-noise ratio (SNR) distribution after making a few reasonable approximations. This model paves a road to approach the evaluation of VANET capacity for safety applications in a more practical and effective way. However, we find out that the model in [1] is not complete in the derivation of capacity, which could hinder the potential model from spreading for the evaluation of practical VANET. In addition to some minor errors and typos, there are issues and limitations in the derivation of node transmission probability and SIR distribution in [1]: 1) The asynchronous timing between the transmission from the tagged vehicle and transmissions from the vehicles that are out of the tagged vehicle's carrier sensing range is not considered in the model. 2) In deriving the probability density function (PDF) of an interferer's distance to the receiver (Eq.(6) in [1]), the probability that the interferer is the l-th node within the right shaded hidden terminal region (Eq.(8) in [1]) is not correct. 3) In the evaluation of interference PDF (Eq.(10) in [1]), impact of possible occurrence cases of interference from two sides of the receiver was not considered in a right way.

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In this letter, in order to build a firm and complete framework of the SIR based approach to the capacity and QoS of VANET for safety message broadcast, we show a new approach to the derivation of the transmission probability and the SIR distribution in the context of the safety applications. The new approach considers the impact of IEEE 802.11p MAC channel access and asynchronous of hidden terminals. Then, the SIR based analysis is further extended to the analysis of QoS metrics and new defined Transmission Capacity for the safety applications.

II. Problem Formulation and Preliminary Results

Given a highway vehicular environment on which all vehicles are equipped with IEEE 802.11p DSRC wireless communication capability, each vehicle broadcasts basic safety message (BSM) containing measured mobility information to all surrounding vehicles in its transmission range periodically with message generation rate λ, and receives the BSMs from the surrounding vehicles. In this way, awareness range of drivers can be extended [2]. The safety-related message broadcast requires high reliability and performance. However, the channel capacity and the QoS is degraded by message collisions and fading in the communication channel.

In this letter, we adopt same communication parameters and same assumptions about vehicular environment and DSRC channel in [1]. To keep the letter self-contained, we briefly summarize the parameters and vehicular environment on which the analysis is carried out: (1) A 2-D strip-like network area can be approximated to a 1-D single lane, as shown in Fig. 1. (2) All nodes are treated as homogeneous with identical vehicle length L_V and transmission power P_t. (3) Denote Y_M as the sum of M neighboring vehicle inter-distance to the tagged vehicle, PDF of Y_M follows the log-normal distribution. (4) *Nakagami* fading model is assumed for vehicular communication channel. Then, the PDF of the

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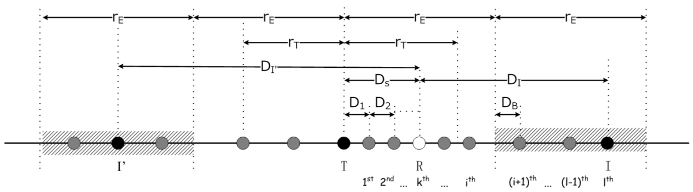

Fig. 1: General interfering scenario for VANET safety message broadcast

power P_r received from a transmitter with distance d away is rewritten as:

$$f_{P_r|d}(x) = \frac{1}{\Gamma(m)}\left(\frac{m}{\bar{P_r}(d)}\right)^m x^{m-1} exp\left(-\frac{mx}{\bar{P_r}(d)}\right) \quad (1)$$

where $\Gamma()$ is the Gamma function, and m is the fading parameter. $\bar{P_r}(d) = P_t\beta\left(\frac{d_0}{d}\right)^{\alpha}$ (d_0 is the reference distance for the far-zone field, α is the pathloss exponent) is the mean value of P_r. (5) The distance between an interferer and the tagged transmitter should be no longer than $2r_E$, where r_E is the average sensing range $r_E = d_0\sqrt[\alpha]{P_t\beta/P_{th}}$,$\beta$ is a transceiver-determined constant, and P_{th} is the clear channel assessment(CCA) sensitivity.

III. New Analysis of SIR Distribution

A. Derivation of Node Transmission Probability

The computation of node transmission probability (p_t) in [1] was referred to reference [9] ([10] in this letter), where p_t was the probability that a node transmits in a slot without considering effect of hidden terminal problem. The impact of asynchronous timing between the transmission from the tagged vehicle and transmissions from the vehicles that are out of the tagged vehicle's carrier sensing range is remarkable, which should be taken into consideration in the analysis.

p_t accounting for the effect of hidden terminal problem can be derived through finding a solution to the Semi-Markov model in [4]. Considering node T and any interferer are out of mutual carrier sensing range, the interferer's transmission could occur at any time of T's transmission. According to [4], the probability that a node in the shaded area transmits during the vulnerable period of the transmission from T is evaluated as:

$$p_t = \pi_{XMT}\frac{2(T_p - DIFS)}{T_p} \quad (2)$$

where T_p is the time duration for one packet transmission, DIFS is a time period of distributed inter-frame space of IEEE 802.11p MAC, π_{XMT} is the steady-state probability that a vehicle is in transmission state, which can be expressed in Eq.(3), σ is time duration of one backoff slot, W_0 is the CSMA backoff window size, ρ is the probability that there are packets in the queue of the tagged vehicle, and q_b is the probability that the channel is detected busy in DIFS time by the tagged vehicle.p_b is the probability that it senses channel busy during one time slot in the backoff process. p_b evaluation has to be reevaluated given the log-normal distribution assumed in [1] instead of Possion node distribution in [4].

$$p_b = 1 - (1 - P_{XMT})^{N_{tr}} \quad (4)$$

$$P_{XMT} = \frac{1}{W_0}\frac{T_p - DIFS + 2\sigma}{T_p}\pi_{XMT} + (1 - \frac{1}{W_0})\frac{2\sigma}{T_p}\pi_{XMT} \quad (5)$$

where P_{XMT} is the probability that a neighbor's transmission is detected by the tagged vehicle in a backoff time slot, and N_{tr} is the number of vehicles in the sensing range of the tagged vehicle, let $N_V = \lfloor r_E/L_V \rfloor$,

$$N_{tr} = \sum\nolimits_{n=1}^{N_V} n \cdot \Pr\{Y_n \leqslant r_E\}\Pr\{Y_n + D_{n+1} > r_E | Y_n \leqslant r_E\} \quad (6)$$

Above equations can be solved by Eq.(3) and Eq.(4) in [1], and by utilizing fixed-point iteration in [4].

B. Distribution of Interference Distance

In deriving $f_{D_I|d_s}(x)$: PDF of interferer's distance to the receiver, the probability p_l that the interferer I is the l-th node within the right shaded hidden terminal region was evaluated as Eq.(7) in [1], which is rewritten as:

$$p_l = \sum\nolimits_{n_r=1}^{N_V} \Pr(N_r = n_r)\Pr(I\ is\ the\ l-th|N_r = n_r) \quad (7)$$

where N_r is the total number of nodes in the right shaded region. The second term $\Pr(I\ is\ the\ l-th|N_r = n_r)$ in [7] is evaluated by Eq.(8) in [1] as $p_t(1-p_t)^{(n_r-1)}/n_r$.

We argue that Eq.(8) in [1] fails to reflect $\Pr(I\ is\ the\ l-th|N_r = n_r)$. Actually, $p_t(1-p_t)^{(n_r-1)}$is the probability that there is an interferer on the right shaded hidden terminal region transmitting on a backoff slot and other nodes in the region do not transmit on the same slot concurrently. It can be used to evaluate if a transmission is successful or not [11]. However, it is different from the required probability that the existing interferer is the $l-th$ node in the right shaded region given $N_r = n_r$. In the broadcast VANET driven by the IEEE 802.11p carrier sense multiple access protocol, once a node gets the channel and starts transmitting, the rest of the nodes in the region will sense the transmission and keep silent even though some of the nodes are ready to transmit. So, $p_t(1-p_t)^{(n_r-1)}/n_r$ cannot catch up the transmission probability under the carrier sensing process case.

Therefore, we divide the analysis of the interference impact into two steps.

First, calculate the conditional probability that the interferer is the $l-th$ node given that $N_r = n_r$ and there exists at least one interferer in the region $[r_E, 2r_E]$ transmits. Assuming all nodes in the region have equal chance, the probability should be revised as:

$$\Pr\{I\ is\ the\ l-th|N_r = n_r\} = \frac{1}{n_r} \quad (8)$$

The probability is independent of p_t (node transmission probability) because the fact that there exists an interferer in the shaded region is implied in definition and derivation of the probability. Then, p_t is evaluated by Eq.(7) and $f_{D_I|d_s}(x)$ is derived from Eq.(7) in [1].

Second, consider the impact of interference existence by p_t on interference power distribution and SIR distribution, which will be analyzed next.

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$$\pi_{XMT} = \frac{2T_p}{[\rho + q_b(1-\rho)][(\sigma + p_b T_p)W_0 + (\sigma - p_b T_p)] + 2T_p + 2(1-\rho)(1/\lambda + DIFS)} \quad (3)$$

C. PDF of SIR

Reflecting on all the factors illustrated above, the evaluation of interference distribution and SIR distribution can be approached by evaluating the CDF and PDF of the interference power received at R from two sides of the receiver separately, then calculate the total interference power accumulated at R considering multiple occurrence cases of possible interferers from two sides of the receiver.

Denote $\Pr\{I\}$ (or $\Pr\{I'\}$) as the probability that at least one interferer I from right side (or I' from left side) transmits when R is receiving the message from T, then

$$\Pr\{I\} = \Pr\{I'\} = 1 - \sum_{n_r=1}^{N_V} [(1-p_t)^{n_r}] \Pr\{N_r = n_r\} \quad (9)$$

where p_t can be derived via Eq.(2).

Hence, considering three possible interference occurrence cases from two sides of the receiver with different probabilities (single interferer from right side, single interferer from left side, and two interferers from both sides), the total interference power $P_\sum$ accumulated at the receiver R is the sum of powers generated from two sides of R with respective probabilities of three cases.

Thus, PDF of $P_\sum$ on R is expressed as:

$$\begin{aligned} f_{P_\sum|d_s}(x) &= \Pr\{I\}\Pr\{I'\} f_{P_{I+I'}|d_s}(x) \\ &+ \Pr\{I\}[1-\Pr\{I'\}] f_{P_I|d_s}(x) \\ &+ \Pr\{I'\}[1-\Pr\{I\}] f_{P_{I'}|d_s}(x) \end{aligned} \quad (10)$$

where the CDF and PDF of interference power $P_I(P_{I'})$ from $I(I')$ given distance $D_s = d_s$ can be evaluated as:

$$F_{P_I|d_s}(x) == \int_{t'=0}^{x} \int_{r_E-d_s}^{2r_E-d_s} f_{P_r|D_I}(t') f_{D_I|d_s}(t) dt dt' \quad (11)$$

$$F_{P_{I'}|d_s}(x) = \int_{t'=0}^{x} \int_{r_E+d_s}^{2r_E+d_s} f_{P_r|D_{I'}}(t') f_{D_{I'}|d_s}(t) dt dt' \quad (12)$$

$$f_{P_I|d_s}(x) = \frac{dF_{P_I|d_s}(x)}{dx}, f_{P_{I'}|d_s}(x) = \frac{dF_{P_{I'}|d_s}(x)}{dx} \quad (13)$$

$$f_{P_{I+I'}|d_s}(x) = \int_0^\infty f_{P_I|d_s}(x-t) f_{P_{I'}|d_s}(t) dt. \quad (14)$$

Given $D_s = d_s$, the SIR at R is the ratio of two random variables ($SIR = P_r/P_\sum$), and its conditional PDF and CDF can be derived by following Eq.(11) and Eq.(12) in [1].

D. Minor errors and typos in [1]

Some minor errors and typos in [1] are listed in footnotes.[1]

IV. EXTENSIONS: QOS AND CAPACITY DERIVATION

Having derived SIR distribution, the following QoS metrics and capacity can be defined and evaluated: Packet Reception Probability, Packet Reception Ratio, and Broadcast Transmission Capacity.

[1](1) $D_{I'}$ range in Fig. 1 is wrong, correct $D_{I'}$ is shown in Fig.1 in this letter. (2) The integration interval for Eq.(9) in [1] for CDF of interference power P_I should be t from $r_E - d_S$ to $2r_E - d_S$ instead of t from 0 to $2r_E$. (3) Definition of Y_M misses specifying $Y_0 = 0$ so that $f_Z(x)$ computation on page 3 is complete ($Y_{l-1} = 0$, as $l = 1$). (4) $F_{SIR}(2^{x/W-1})$ in Eq.(13) should be $F_{SIR}(2^{x/W} - 1)$.

A. QoS Derivation

Message (packet) reception probability (PRP) is defined as the probability that a receiver successfully decodes the message (packet) from a source node with a distance. We observe that, in real wireless communication system, it is possible that SIR is bigger than the specified threshold, but the received signal is too weak to decode. So, we point out that pure SIR based signal receiving is not sufficient for making decision on successful transmissions. The probability that receiver with distance d_s to the tagged node accepts the message successfully if the measured conditional SIR is higher than the given threshold and the received signal is stronger than the reception threshold or CCA P_{th}. Assuming the two conditions are independent of each other, the probability is expressed as:

$$\begin{aligned} PRP(d_s,\theta) &= \Pr(SIR \ge \theta|d_s) \cdot \Pr(P_r \ge P_{th}|d_s) \\ &= (1 - F_{SIR|d_s}(\theta))(1 - \int_0^{P_{th}} f_{P_r|d_s}(x)dx), d_s \le d_{ROI} \end{aligned} \quad (15)$$

Define region of interest (ROI) of a safety application as size of the geographical region covered by those entities participating in the application, which is denoted as d_{ROI}. Different safety applications have different ROI sizes [8].

Then, packet reception ratio (PRR), defined as the percentage of receivers within ROI that are free from transmission errors once a message is sent out, can be evaluated as:

$$PRR(d,\theta) = \int_0^d PRP(x,\theta) f_{D_s}(x) dx, d \le d_{ROI} \quad (16)$$

where $f_{D_s}(x)$ can be derived from Eq.(2) in [1].

B. Definition and Evaluation of Transmission Capacity

It is very hard to directly apply the channel capacity to the design of VANET for safety applications. Recently, a new concept of transmission capacity for one-hop wireless ad hoc networks has been defined and evaluated [12] [13]. Here, we define Transmission Capacity in the context of broadcast VANET for safety applications as the maximum overall message generation rate multiplying with the number of vehicles within ROI of the safety event, subject to a constraint on PRR, which can be formulated as:

$$\begin{aligned} &C_T = Max\{N_{ROI}\lambda\} \\ &subject\ to\ PRR(\lambda,d,\theta) \ge \xi_p(), d \le d_{ROI} \end{aligned} \quad (17)$$

where ξ_p is the reliability requirement, and N_{ROI} is the average number of vehicles within ROI, which is evaluated as:

$$\begin{aligned} N_{ROI} = \sum_{n=1}^{\lfloor d_{ROI}/L_V \rfloor} & n \cdot \Pr\{Y_n \le d_{ROI}\} \\ & \Pr\{Y_n + D_{n+1} > d_{ROI}|Y_n \le d_{ROI}\} \end{aligned} \quad (18)$$

The transmission capacity allows the VANET designers to see exact dependence between QoS and the possible design choices and network parameters. Since $PRR()$ is monotonically decreased with λ value as the other network parameters

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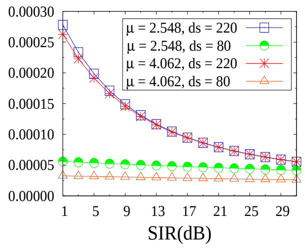

Fig. 2: Conditional PDF of SIR at receiver

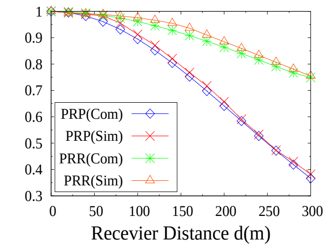

Fig. 3: PRP and PRR of VANET with NS2 simulation (μ = 2.548, $\lambda = 10pkts/s$)

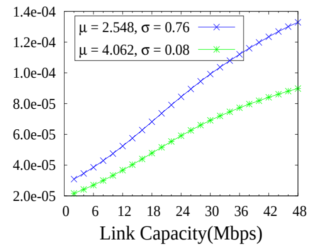

Fig. 4: PDF of Link capacity of VANET broadcast

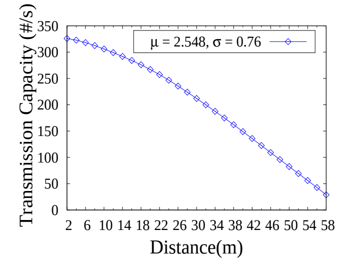

Fig. 5: Transmission Capacity with $d_{ROI} = 50m$ and $\xi_p = 0.99$

are fixed, the capacity C_T can be recursively derived within certain error from a random initial value of λ so that $PRRs$ with new λ meet the defined QoS requirements.

V. Numerical Results and Discussions

To validate the new proposed theoretical analysis, Matlab (for 802.11p p_t evaluation) and Python (for distributions and QoS metrics computation) are deployed for theoretical computations and NS2 is deployed for network simulations. A VANET in a highway where vehicles are distributed according to log-normal distribution is set up for both theoretical computation and simulations. Each vehicle in the VANET is equipped with DSRC capacity. We adopt the same communication network parameters in [1] with packet generation rate 1~10 packets per second (packet length = $200bytes$), data rate = $24Mbps$, PHY preamble + PLCP header $T_{H1} = 44\mu s$, MAC header $T_{H2} = 272bits$, and practical *Nakagami* fading parameters ($m = 1$).

Fig. 2 shows the conditional PDF of SIR at the receivers with different values of μ in the car-following model, the signal propagation distance d_s, and SIR thresholds. It can be seen from Fig. 2 that the PDFs increase with the propagation distance but decrease with the SIR threshold. Also, if the expected safe distance between moving vehicles from 13 meters ($\mu = 2.548$) to 100 meters ($\mu = 4.062$), the PDF values decrease accordingly.

Fig. 3 shows the *PRP* and *PRR* at the receivers with different values of the signal propagation distance d (*SIR* threshold θ = 30dB, $P_{th} = 0.0028Watts$). It is shown from Fig. 3 that the analytical results practically coincide with the simulation results, which verify correctness of the proposed model. Both *PRP* and *PRR* significantly decrease with the propagation distance.

Fig. 4 depicts the PDF of link capacity [1] of a randomly selected communication pair in the local VANET for BSM broadcast. Given that the link capacity is less than $50Mbps$, the higher density of vehicles on the road, the bigger the PDF value is.

Fig. 5 demonstrates the new derived transmission capacity (or maximum message generation rate) given *PRR* requirement ξ_p=0.99, region of interest for rear-end collision $d_{ROI} = 50m$. Since increasing message generation rate adds more interference to each receiver, the transmission capacity C_T decreases with communication distance and density of vehicles.

VI. Conclusions

This letter reconsider the SIR based capacity approach in [1]. The issues in [1] have been pointed out and fixed. Then, the new model is extended to evaluate new QoS metrics and defined transmission capacity. The future work would be applying the model for analysis and design of 2-D VANET safety applications and spectrum sharing of heterogeneous VANETs.

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